

# Geometry of Time, Axiom of Choice and Neuro-Biological Quantum Zeno Effect

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## Abstract

Role of axiom of choice in quantum measurement is highlighted by suggesting that the conscious observer chooses the outcome from a mixed state. Further, in a periodically repeating universe, these outcomes must be pre-recorded within the non-physical conscious observers, which precludes free will. Free will however exists in a universe with open time, It is suggested that psychology's binding problem is connected with Cantor's original definition of set. Influence of consciousness on material outcome through quantum processes is discussed and interesting constraints derived. For example, it is predicted that quantum mechanical brain states should get frozen if monitored at sufficiently small space-time intervals - a neuro-biological version of the so called quantum zeno effect, which has been verified in domain of micro-physics. Existence of a very small micro-mini-black-hole in brain is predicted as a space-time structural interface between consciousness and brain, whose vaporization explains mass-loss reported in weighing experiments, conducting during the moments of death.

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# 1 Introduction

Consciousness is probably the most difficult problem attempted by human scientific endeavor, and is developing into an eclectic discipline. In this paper, I introduce certain new set theoretic ideas (among others) in the already inter-disciplinary field of consciousness studies. At the outset, I shall clearly state relevant points of my philosophical stance.

## A. Nature of Consciousness.

Conscious observers are different from brain and bodies, but interact through them. This is identical to the dualistic school of thought, with Eccles [1] as a representative. This apriori does not rule out emergence of temporary consciousness in matter, as a result of various postulated mechanisms [2], such as Bose-Einstein condensations, or self organizing behavior, or phase locked dynamical neural networks, strange attractors etc.. Whereas the material consciousness is temporary (depending upon stability of the physical system), the non-material observer is eternal in time. This does not imply that observer is always conscious, or observer is conscious only in body. The term, 'Soul', is more accurate, as consciousness and observation are 'temporary phenomena' accompanying 'Soul', in certain conditions <sup>1</sup>. I shall however continue to use the term, "conscious observer", to represent soul. Consciousness may thus be regarded as a common emergent property of both the observer, as well as matter. As can be seen, the stance is flexible, enough to be compatible with almost diverging views on the subject, and may be termed as 'Non-dualistic' in the sense that it allows simultaneous existence of almost divergent view points. Further, the conscious observers are distinct from each other, and inhabit the same (one) physical universe.

## B. Nature of Time.

It is probably unlikely that complete problem of consciousness can be solved without understanding nature of time. Time has been a subject of many monographs and papers [3, 4, 5] in physics. The unresolved issues here are -

1. Arrow of Time, i.e., its irreversibility,
2. Origin of present moment, alternatively the observed subjective distinction between, past, present and future, with conscious observers'

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<sup>1</sup>For instance, during dreamless sleep and coma, both consciousness and observations are absent

attention confined to present (light-like hyper-surface), (ignoring out of body and similar psychic experiences [7] , for the time-being).

3. Overall geometry of time, i.e., cyclic or linear etc., [8].

In periodic time, it can be shown that cause and effect are connected by, what in set theory is called an equivalence relation [9]. However, this causal structure could be important, as it has been pointed out that EPR paradox [38], which has been experimentally verified in recent years [11, 12, 13, 14, 15, 16], does suggest that causality is an equivalence relationship [17]. While this circular geometry of time, would appear to lead to the usual causal paradoxes of time loops in physics, such as killing one's mother before one's birth (or conception!) - the causal paradoxes are avoided by withholding free-will to the conscious observers or actors or participants (in the time cycle!). Thus, consciousness is only an observer of events pre-recorded within itself, in an over all cyclic time, within this philosophical framework. Any apparent freewill is actually illusory.

Further cyclic nature of time blurs the distinction between past and future, as the two are globally connected. In introduction of his monograph, Zeh [3] quotes Lewis Carroll from the book "Through the Looking Glass" -

White queen to Alice: *"Its a bad memory which works only backwards"*.

In cyclic time, a good or perfect memory, will be able to remember all events which the possessor (conscious observer) would have observed with in the complete time cycle.

I should add that while cyclic time has been a hallmark of ancient cosmological systems, Poincare , Zermelo, Caratheodoty, and Nietzsche [18, 19, 20, 21] did attempt a mathematical formulation of the idea at the turn of the century, and the idea has recently been evoked by contemporary mathematical physicists such as Segal [23], Guillemin [22] and others [24, 25] , with a view of solving certain problems of observational cosmological physics, and particle physics (macro and micro cosmos). Idea behind introducing cyclic time concept is that if problem of consciousness is going to be solved [or the other way round), it may be so only in cyclic time - or what the mathematical physicists would call the  $S^1$  (circle) topology of time.

## 2 Set Theoretic Connection.

Cantor defined set as "collection into a whole, of objects of our intuition or thought". The definition is very psychological, and the phrase "collection into a whole" is of special relevance. The phrase actually implies simultaneous perception of constituent set elements as a whole. Even a sentence is understood only when perceived as a whole. Role of short term memory in verbal comprehension comes to mind when perceiving or comprehending very large sentences - here  $7 \pm 2$  chunks of short term memory are probably being used at various levels, and the sentence has to be read many times, before comprehension (collection into a whole) occurs. Interestingly  $7 \pm 2$  has been derived by statistical mechanical considerations, by developing so called Fokker-Plank equation of brain neurons [26]. I equate verbal comprehension with (verbal) perception, as such an equality is the reason for use of phrases such as "Now I see", when verbal comprehension occurs.

While verbal perception occurring in sentence comprehension involves only a small number of elements, visual perception by contrast requires integration of a very large number of elements. Same can be said of other modes of perception, such as auditory, tactile, kinesthetic, olfactory, proprioceptive. Further, in the conscious experience of an observer, these various perceptions from different senses, are further integrated into a whole, which even has non-sensory components such as thoughts, memories, feelings, emotions. The complete momentary experience of a conscious observer is thus a set, in context of Cantor's original definition. Further its a finite set, as it has a finite number of elements, as represented by finite number of brain neurons.

## 3 Quantum Mechanical Aspects

von Neumann [27], Wigner [28] and Pauli [29] have suggested that wave-packet reduction was occurring when the wave-packet was interacting with the conscious observer. Precise mechanism for this reduction by interaction with consciousness has never been worked out. On the other hand certain mechanisms for wave-packet reduction have indeed been worked out by physicists, e.g., wave-packet reduction occurs when a coherent (unreduced) wave-packet approaches a system with infinite degrees of freedom [30, 31]. Hawking [?] has also obtained interesting results concerning wave-packet reduction near a black-hole when a coherent state approaches it.

There exist other interpretations of quantum mechanics which try to do away with concept of wave-packet reduction all together. These include Everett's many world interpretation [33] , Bohm's interpretation Bohm 1957, various hidden variable scenarios Bellifante 1973 etc.. However, experimental evidence of quantum zeno effect Itano 1990, and EPR suggests that wave-packet reduction is a good concept to explain the results. I therefore assume that wave-packet reduction does occur, and further it is caused by conscious observer. What are the properties of consciousness which solves these quantum measurement problem? Let's refer to this set of properties of consciousness as OM ( O - Observer, M - Measurement). Acronym OM has been selected, because of special significance word 'Om' has in context of search for one's true identity in ancient Indian philosophy.

von Neumann's model can be understood in terms of "Quantum Measurement Chain" (QMC). In this when an attempt is made to record state of say Schrodinger's cat, by a camera, the wave function of camera also passes into a coherent state. Same happens to the wave functions of film, human eyes, retina, brain neurons and so on. In von Neumann's interpretation, observer lies at the end of this quantum measurement chain, and leads to wave packet reduction of all the intermediary links (camera, brain, neurons) in this quantum measurement chain. This chain is originating at Schrodinger's cat. Its possible for more than one QMC to originate from the same system, e.g., when multiple observers are monitoring state of Schrodinger's cat. Such quantum measurement chains will be called linked.

Now, apriori there is no reason, why if consciousness is causing wave-packet reduction (and influencing physical universe), it is not through the two mechanisms already outlined by physicists Hepp, Fakuda, and Hawking (above). In absence of any other description of process of consciousness causing wave-packet reduction, I accept what is not forbidden. Thus, as a conceptual agent responsible for wave-packet reduction, I attribute following properties to consciousness -

**OM-1.** Consciousness is a system of infinite degrees of freedom.

**OM-2.** Embodied Consciousness is associated with a black-hole.

Reason for use of letters **O** and **M** in **OM** will become clear in sections 4 and 5.

## 4 Black-hole in brain!

While popular notion of black-holes as massive astro-physical objects still awaits experimental confirmation, theoretical physics has moved ahead with concept of mini-black-holes, which are much less massive, and those which can be light enough to have mass of elementary particles, such as protons [37, 38]. While the astro-physical black-holes are caused by gravitational collapse of stars whose mass exceeds the so called Chandrasekhar limit, the concept of micro-mini-black-hole (MMBH) is more like a singularity or hole in fabric of space, i.e., its a place, where physical space ceases to exist, so to say. Its gravitational influence is negligible, being proportional to its mass, and so is its size. Its interesting because of its relativistic, quantum mechanical, and philosophical properties. The reader thus need not be alarmed, that all of his or her gray matter will be sucked down this infernal black-hole in brain.

There is an interesting philosophical reason for existence of a black-hole associated with consciousness in brain. While the soul or conscious observer, is regarded as a non-physical object, non-localizable in space-time, the present brain studies have almost localized it to be the region within brain. Black-hole provides a escape for the non-material spirit. While the black-hole can be given physical co-ordinates, the area within the event horizon, of the black-hole, effectively does not belong to the physical universe - it lies beyond the physical, universe so to say. Thus conscious observer located within such a black-hole strictly speaking does not exist within physical space-time. In the event of physical death, the mind-body connection is severed, (e.g. Moody [39] ), and one expects this black-hole to dissolve, or evaporate by a mechanism, analogous to "Hawking radiation" [40]. Its energy will be carried away by gravitational waves, and therefore will lead to a mass loss equal to mass of MMBH (few grams). Also effect of these gravitational waves generated at the moment of death, will be similar to high frequency acoustic waves, and would lead to cracking of any glass enclosure containing the physical body. Experiments verifying these phenomena have actually been done at the turn of this century [41]. The kind of tunnel vision reported in near death experiences, i.e., motion through a long dark tunnel, with light at the end of tunnel [39], is actually in accordance with optics near black-holes - an observer escaping through a black-hole would actually experience, similar tunnel vision!

Black-holes have many other interesting properties such as existence of local closed time like curves, Morris 1988 which could explain ability of clair-

voyants to see past and future events, while existing within the physical body (by motion of point like conscious observer, on one of the local closed time like curves near MMBH). Black holes also provide a handle, or gateway to other dimension, and non-physical universes, which are of special interest Brahma Kumaris and possibly other workers in Transcendental. Interestingly, tachyons (particles traveling faster than light) falling through a black-hole, leave the conventional physical universe of three space and one time dimension, and enter a universe with three time and one space dimension Chandola 1986. This latter universe or rather meta-universe, could be of special interest for actual meta-physical experiences.

## 5 Records within Consciousness

Apriori, von Neumann's interpretation of consciousness causing wave-packet reduction, does not determine, as to which particular outcome is actually selected. Neither do any other mechanisms such as Fakuda's, Hepp's or Hawking's - all they do is reduce a coherent superposition of states into an incoherent mixture - actual outcome then being a psychological process of observation. Thus if embodied consciousness is monitoring quantum states of 109 neurons, and causing their wave-packet reductions, which leads to perception, and the complete momentary experience of that observer, then these wave-packet reductions must be recorded within the conscious observer, in a consistent fashion, and cannot be random because - random reductions, will not lead to the perceived order of the universe. Various laws of physics, such as those of continuity, conservation, invariances, etc., are result of perceptions, based upon these wave-packet reductions. Hence we can identity another property of consciousness -

**OM-3.** Recorded within conscious observer is outcome of all quantum measurements performed by it (as reflected in coherent brain states). These reductions are not random, but have a logical relation to each other, which is the basis for invariances observed in physics, and results of EPR and Bell's inequality experiments etc.

Now this selection of a particular outcome, from the set of all possible outcomes is another psychological process, which was encountered quite early ;in development of set theory. Its called Axiom of Choice [44]. Briefly, it states that, given a set, there exists a choice function, which selects an element of the set. Only problem is, that while it is vital to almost all of mathematics,

its use has lead to paradoxical Banach-Tarski theorems, involving duplication of spheres, and show that concept of additive measure is not sound [45]. Details of how axiom of choice relates to quantum measurement process are worked out in [?]

There exists another reason for records within consciousness, and evidence from it comes from following scenario reminiscent of EPR of quantum mechanics. The argument has a philosophical flavor - if distinct observers A and B separated by a space-like interval, observe state of Schrodinger's cat in an experiment at a particular instant, it is required that both should observe it either dead or alive. It should not be that observer A, finds cat alive, and observer B finds cat dead. Thus if wave function of Schrodinger's cat as represented in brain neurons of observer A is collapsing, due to a recording within A, this collapse is compatible with a similar collapse occurring in brain of observer B. To ensure this compatibility we require -

**OM-4.** Recording within conscious observers with respect to measurement performed on the same quantum system are mutually compatible. Alternatively, outcomes observed by observers lying at ends of distinct but linked quantum measurement chains, are compatible.

Support of EPR results of quantum mechanics for records within consciousness is as follows. Lets say that observers A and B are separated by a space-like interval, and perform measurement on a correlated photon pair. EPR results indicate that wave functions of both the photons are collapsing only at the moment of measurement, and the collapses are mutually compatible, which A and B will also notice, when they compare notes latter on. Now, there exists no way for observer A to send a signal to B, regarding his or her outcome, within the frame work of present day physics (light speed limit and all that). The question therefore exists, if the conscious observers A and B are indeed causing these distinct but correlated collapses, how is mutual compatibility being ensured? OM3 is thus related to OM4. This is also where cyclic nature of time may be playing an important role. In cyclic time, the same measurements would have been performed in all the past (infinite) time cycles, and identical outcomes would have been recorded, in all the time cycles. This geometry of time, appears to provide at least a chance for observers to compare note and correlate their outcomes. See [9] for a possible scenario of communication between observers in EPR type experiments using light like signals in cyclic time.

I close this section, with an argument, for why the embodied consciousness should exist in a MMBH (micro-mini-black-hole). If outcomes of quantum

measurement are pre-recorded in the conscious observers, (OM-3 and OM-4) such a recording constitutes "hidden variable [35] determining the quantum measurement outcome". Now Bell's theorem [47] yielded inequalities, which would distinguish between hidden variable scenario, and actual quantum mechanics (without hidden variables). Experiments [11]-[16] to test between the two yielded results in accordance with quantum mechanics, i.e., no functions or additional physical hidden variables, which would determine the outcome. Locating the consciousness within a black-hole resolves this problem, because now the recording (hidden variable determining the outcome) is lying beyond the event horizon of the black-hole, and thus effectively outside the universe, and therefore is beyond the purview of present formulation of Bell's theorem [47].

## 6 Neuro-Biological-Quantum-Zeno-Effect

Readers would be familiar with ancient Greek Zeno's paradox [49], which questioned the concept of motion, by arguing that if an arrow in flight was being continuously observed, and occupied a position at every time instant, as to how could the apparent motion observed was actually possible? Though the paradox was resolved in continuum based classical mechanics, it has reappeared in grab of quantum zeno effect (QZE) - so christened by physicist Sudarshan [49]. Briefly, if a system is in state A, and about to change to state B, before it does so, its wave function (mathematical object describing its physical state), has to go into a superposition of states A and B, i.e., the system exists in a sort of (A+B) state. Now when a quantum measurement is performed on this superposed (A+B) state, the wave function collapses to either A or B. So, if the measurements are being performed sufficiently rapidly, the wave function of the system cannot evolve to first state (A+B) and than state B. As a result it remains frozen in state A. This effect is also called "watch dog effect", [50] (thief moves only when watch dog closes its eyes), and the "boiled kettle phenomena", (kettle appears to boil over and spill, just when one's attention is diverted). Thus in terms of quantum mechanics, paradox of Zeno's arrow can be formulated and resolved as follows. Where as, wave-function of arrow, which also describes its position is evolving continuously, the actual act of wave-packet reduction, by monitoring or perception of a conscious observer, is a non-continuos phenomena. This is because the process of human perception requires a large number of photons

from Zeno's arrow to reach human eye and retina, where after a time delay, a signal is relayed to brain, and a quantum mechanical representation of arrow's (superposed) wave function is formed by observer's neurons. This quantum measurement chain of coherent (superposed) state collapses when the conscious observer perceives arrow's position, and is a non-continuous phenomena. Thus, in between these non-continuous perceptions, wave function of arrow can evolve to different positions. QZE has been verified for ensembles of atoms about to make electron transitions from a higher energy state to a lower energy state [36]. Monitoring at progressively smaller intervals, reduces actual number of atoms making the transition, in a given period of time.

Neuro-biological-quantum-zeno-effect (NBQZE), as the term suggests, implies that, if brain state of a person is being monitored at sufficiently small space-time scales, (by another person, with data being recorded onto say a computer, all of which is later examined by the experimenter), then neurons of the subject will not be able to evolve to a coherent state and make transition, to another state which would represent transitions from one perception to another. Thus person's subjective experience would be blocked. Even though external sensory stimulus may be applied, the subject would not report perception of the stimulus. The resultant state would be similar to highest states of meditation, which involve complete withdrawal of consciousness from the body and senses - effectively the consciousness has ceased to interact with the physical universe, and is no longer performing any quantum measurement - wave-packet reductions, in his or her brain are being caused by the experimenter, and are preventing brain state from evolving along with the wave function of the changing environment or universe.

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