

Recurrence Metrics and the Physics of Closed Time-like Curves

M.S.Modgil* Deshdeep Sahdev†

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Department of Physics, Indian Institute of Technology, Kanpur-208016, India

Abstract

We investigate vacuum solutions of Einstein's equations for a universe with an S^1 topology of time. Such a universe behaves like a time-machine and has geodesics which coincide with closed time-like curves (CTCs). A system evolving along a CTC experiences the Loschmidt velocity reversion and undergoes a recurrence commensurate with the universal period. We indicate why this universe is free of the causality paradoxes, usually associated with CTCs.

1 Introduction

The steady proliferation of solutions containing CTCs suggests that closure in time is more generic than esoteric in the General Theory of Relativity (GTR). Indeed, following Godel's [1] discovery of a universe containing CTCs, these were found to exist in several other space-times as well, one of which in fact predated Godel's: In 1974, Tipler [2] showed that van Stockum's [3] 1936 solution of Einstein's equations, for a rapidly rotating infinite cylinder, had CTCs (noted earlier, for the interior, by Maitra [4]) which allowed any two points of this space-time to be causally connected. CTCs were subsequently also found in the Taub-NUT [5], Kerr [6] and Tomimatsu-Sato [7] solutions, in worm-hole space-times [8] and in the neighbourhood of oppositely moving

*msingh@iitk.ac.in

†ds@iitk.ac.in

cosmic strings [9]. CTCs have also been discovered by Li [10] and Everett [11].

In view of this, it is difficult to maintain that the corresponding space-times are pathological, much as any attempt to make physical sense of them brings up a host of fundamental (and vexed) questions of interpretation. The most important of these are the issues of causality violation and non-unitary evolution, to which several authors have reacted by arguing vigorously that in *no* space-time can CTCs be actually *generated*: they can be at best be *already* present [12].

The consistency of quantum field theories defined on such space-times has also generated some debate. Thus, while the Cauchy problem for the scalar wave equation is well-defined on wormhole space-times, [13] it was pointed out that vacuum polarization effects produce a divergence, near the Cauchy horizon, in the renormalized stress-energy tensor of the scalar [14] field, but the divergence then turned out to have nothing to do with CTCs [15].

Finally, quantum mechanical versions of time machines seem to fit in well with the many-universe interpretation of quantum mechanics [16, 17] while path integrals generalized to include those on the CTCs of a time-machine show that, for unitary evolution, the time machine must be causally isolated from the rest of the Universe [18].

Leaving aside for the moment, the bigger question of what happens if a region containing CTCs, is *embedded* in a Universe with an otherwise R^1 -topology of time, let us turn to what exactly happens to a system which circumnavigates a CTC (in the interior of a time-machine, for instance)? Since none of the CTCs discovered so far coincides with a geodesic of the underlying space-time, motion along these occurs only under the influence of external (i.e. non-gravitational) forces. For the Godel space-time, for example, it requires very high accelerations and large time-periods to return to one's past [19].

The need for external forces can, however, be obviated if we are willing to turn the entire Universe into a time-machine, i.e. to consider the time-dimension itself to be closed. CTCs then coincide with geodesics and the discussion can be carried out entirely in terms of gravitational free-fall. Furthermore, since nothing but the time-machine now exists, the question of isolation is inapplicable and we can expect unitarity to hold unconditionally.

Another major gain of this approach is simplicity. Recall that for the quantum mechanical particle inside a box, the solution of the governing Schrodinger equation is immediate. All one has to under-

stand the effect of the boundary conditions. Likewise, the simplest Universe with S^1 -time is the S^1 analog of Minkowski space. But the solution contains some non-trivial physics (due to constraints imposed by closure in time) and much of this applies to CTCs in general.

We note, parenthetically, that while the topology of the time dimension, for the Universe we live in, is *locally* known to be R^1 , *globally*, it is actually more logical to assume it to be that of a circle whose radius must be constrained by observation. This would be completely analogous to our pinning down the spatial curvature of our Universe by measuring its density and comparing it to the critical density required for closure. It is, thus, surprising that the closest anyone has come to doing this is Segal, who developed a model of the Universe in which future and past infinity were identified, and in which the standard cosmological observations found alternative explanations [20].

The rest of the paper is organized as follows. Since the boundary conditions contain much of the physics, Section II is entirely devoted to a discussion of the time-closure constraint. In Section III, we discuss two different vacuum solutions, a time-independent pedagogical one in III(a), and a class of time-dependent ones in III(b). In Section IV, all the insights gained are put together and discussed in the context of CTCs in general.

2 The Time-Closure Constraint

A Lorentzian manifold with an S^1 topology of time is periodic in the sense that every event has an infinite number of copies in both past and future. In such a universe, *all* particle paths — geodesic as well as non-geodesic — close in an identical period and the initial Cauchy data on a space-like hypersurface recurs an infinite number of times.

To understand what this means, consider the manifolds [21], S^n and RP^n (i.e. S^n with antipodal points identified), *all* of whose geodesics close in the same proper distance — $2\pi R$ and πR respectively, where R is the radius of the sphere. Consider further an arbitrary distribution of (non-interacting) photons in these spaces, and take the photons to be moving along geodesics. Each photon will clearly circumnavigate the great circle/semi-great circle it happens to be on, in the same time period, $T(= 2\pi R/c$ or $\pi R/c)$. As a result, the spatial distribution of photons at time T , will be identical to the one we started with. Note that for this to work, all particles in the distri-

bution have to be moving with the same speed. Thus if, in addition to photons, the distribution contains material particles, recurrence by circumnavigation will *not* occur. It follows that $S^3 \times R^1$ (S^3 -space, R^1 -time) and $P^3 \times R^1$ are 4-dimensional Lorentzian manifolds, all of whose null geodesics are periodic. (Such space-times have been studied in the literature [22], and are said to have Zollfrei metrics).¹ In the purely light-like sector, they are therefore equivalent to $S^3 \times S^1$ (S^3 -space, S^1 -time) and $P^3 \times S^1$ respectively.²

We turn next to a mechanism which *can* bring about the equiperiodic closure of time-like geodesics in Lorentzian manifolds. The key to this mechanism is provided by the recurrence (i.e., the re-emergence of arbitrarily specified initial data on a *distinct* space-like hypersurface [25]), which this closure must inevitably produce. We emphasize that we are talking about an *exact* recurrence, occurring in a *finite* proper time, and *not* about the arbitrarily close, asymptotic recurrence which Poincare discussed in the context of classical ergodic systems.

Recall that what Poincare proved was this [26]: Given a set M of points, and its σ -algebra \mathcal{M} of subsets, let $x \in C \in \mathcal{M}$, and let \mathcal{T} be a measure-preserving transformation group of M parameterized by a real and continuous s . (For a statistical system, M would represent the phase space of configurations; x , a specific configuration, and \mathcal{T} , the group of flows parameterized by time s). Then, there exists an s , such that $\mathcal{T}_s(x) \in C$, for any $C \in \mathcal{M}$ and $x \in C$. In other words, *given sufficient time*, the system will return *arbitrarily close* to its initial state, and will do so infinitely often. The recurrence is clearly not exact and the theorem does not, *in any way*, limit the amount of time it takes the system to return to an arbitrarily specified neighbourhood of the initial state.

Loschmidt, on the other hand, pointed out that an *exact* recurrence *would* occur, in a thermodynamic system, if each and every particle of the system underwent a *simultaneous* velocity reversion, i.e, the momentum, \vec{p} , of *every* particle changed to $-\vec{p}$ on a single time slice; put another way, at a given instant of time, every particle (photons inclusive) encountered an invisible, infinitely massive wall, at normal incidence, and suffered a reflection. Indeed, once such a velocity reversion occurs, every particle retraces its path in configu-

¹Manifolds with Zollfrei metrics automatically regularize the infra-red divergences of field theories defined on them [23, 24].

² $S^3 \times S^1$ was the basis of Segal's cosmological model [20].

ration space, and re-enacts each collision it had earlier suffered — in reverse chronological order: A film of the microscopic evolution of the system simply starts running backwards from the moment of reversion, the time-reversed evolution being perfectly permissible because Newtonian dynamics is invariant under time-reversal.

We note that while a single reversion is enough to make the system retrace its path in configuration space, *two* reversions are required to make the phase space trajectory as a whole, periodic. At each reversion, the point representing the system’s configuration in phase space, behaves as follows: Its projection, on the subspace of momenta, jumps discontinuously while in the (configuration) subspace of spatial positions, its projection continuously backtracks. Furthermore, if two such reversions occur with a given periodicity, the system executes a cycle, limited at each end by the imaginary walls producing the reversions. An inverse \mathcal{T}_s^{-1} [27] can then be defined for every flow \mathcal{T}_t (and a group character can thereby be conferred on \mathcal{T}). Indeed, letting \mathcal{C} denote Cauchy data at time t , $\mathcal{T}_t(\mathcal{C}(0)) = \mathcal{C}(t)$. Since, $\mathcal{C}(0) = \mathcal{C}(T) = \mathcal{T}_T(\mathcal{C}(0))$, it follows that $\mathcal{T}_T = Id$. Now, $\mathcal{T}_T = \mathcal{T}_{T+t-t} = \mathcal{T}_t\mathcal{T}_{T-t}, \Rightarrow \mathcal{T}_t^{-1} = \mathcal{T}_{T-t}$.

For a *classical* thermodynamic system, there is, however, no mechanism for effecting such reversion in velocity and Loschmidt’s observation has thus remained a mere curiosity. In GTR, on the other hand, such a mechanism *does* exist, as we show in detail below. The reason is that here, particle systems are tied to geodesics which, in turn, are determined by the underlying metric. If this happens to be a solution to the Einstein field equations on a manifold with S^1 time, recurrence is enforced by the metric itself, i.e. the corresponding geodesics automatically incorporate the Loschmidt velocity reversion.

Explained differently, S^1 -time introduces an effective velocity-dependent potential which reverses the trajectories of all particles on a single time slice. The situation is somewhat analogous to that of a particle thrown up vertically in the earth’s gravitational field (with less than the escape velocity): The particle eventually stops, reverses and reaccelerates downwards. In doing so, it revisits all the positions it had previously occupied, at the corresponding speeds, before reaching its starting point. The time of reversion in this case is however dependent on the initial velocity, in contrast to what is implemented by S^1 -time.

Two remarks are in order at this point. Firstly, the above result does not, in any way, contradict the ‘no return’ theorem [25], proved by Tipler, for a closed universe, beginning and ending with a singu-

larity, simply because the premises of that theorem do not hold for a universe with S^1 -time. Secondly, since a particle's position in GTR is represented by a 4-vector, one of whose components is time, we intuitively expect that the backtracking in space brought about by (4-)velocity reversion, would in general be accompanied by backtracking in time, i.e. by time-reversal. In other words, we expect the discussion to be closely tied to the circumnavigation of a CTC, as indeed it is.

To see the time reversal explicitly, we first note that there are 2 distinct times in the problem: The coordinate time, t , and the proper time, τ . The former appears in the metric and takes on all values on the real line. It wraps around the S^1 of time once, each time it increases by T , the period of recurrence. The latter, on the other hand, is the time kept by a clock comoving with a particle inside the periodic universe. It is *this* which displays the reversal we are discussing.

The two times can be simply related by setting all spatial separations in the line element to 0 and identifying the proper distance with the proper time to get $d\tau = \sqrt{g_{00}}dt$. Now for a universe with S^1 -time, g_{00} is necessarily periodic and smooth in t , which makes τ likewise so. But then τ must have 2 (or, more generally, an even number ³ of) turning points in the interval $[0, T]$. This, in turn, implies that every time-cycle *must* have a region of time-reversal where $d\tau/dt < 0$, i.e. τ decreases monotonically, and *all* processes reverse direction, bringing about, among other things, a decrease in entropy. It is, incidentally, the existence of this region that enables *all* physical clocks to display the same readings at coordinate times, $t = 0$ and $t = T$.

Interestingly, this time-reversal seems to be completely generic. Its occurrence has been *individually* noted for each of the space-times in which CTCs have been found to exist. We shall return to this point in our concluding discussion.

3 Some Vacuum Solutions

We now examine some explicit examples of recurrence metrics satisfying the vacuum Einstein equations. The first of these has the virtue of being mathematically trivial, and yet capturing the physical essence of the problem.

³ Cf Milnor [29] for a proof based on Morse Theory of the following result: A continuous function f defined on a closed curve Γ parametrized by s , has an even number of critical points where $df/ds = 0$.

3.1 The Periodic Minkowski Space-time

Consider the following modification of the Minkowski metric:

$$ds^2 = c^2 d[F_T(t)]^2 - dx_1^2 - dx_2^2 - dx_3^2, \quad (1)$$

where the function $F_T(t)$

- is periodic in $[-T/2, T/2]$, and
- satisfies $\lim_{T \rightarrow \infty} F_T(t) = t$,

a concrete example being

$$F_T(t) = \frac{T}{2\pi} \sin \frac{2\pi t}{T}. \quad (2)$$

The Minkowski space-time (or, more precisely, the union of this and the point at infinity) can be recovered as a limiting case of this (periodic) universe: We simply let the recurrence period $T \rightarrow \infty$, and note that $F_T(t)$ must then coincide identically with t .

The only non-zero Christoffel symbol corresponding to the above metric is

$$\Gamma_{00}^0 = \ddot{F}_T(t) / \dot{F}_T(t). \quad (3)$$

This does not affect, in any way, the Riemann curvature tensor: All components of the latter vanish identically, as for the Minkowski metric. The periodic Minkowski space-time is thus flat and constitutes a vacuum solution of the Einstein Field Equations.

Γ_{00}^0 does however alter the geodesic equations, which are now given by

$$\begin{aligned} t''(s) - t'(s) \frac{\ddot{F}_T(t)}{\dot{F}_T(t)} &= 0, \\ x''(s) &= 0, \quad i = 1, 2, 3 \end{aligned} \quad (4)$$

where primes and dots denote differentiations with respect to the geodesic parameter, s , and the time, t , respectively.

These equations readily integrate to

$$\begin{aligned} t &= F^{-1}[c_1^0(s - c_2^0)], \\ x_i &= c_1^i s + c_2^i, \quad i = 1, 2, 3, \end{aligned} \quad (5)$$

where c_1^μ and c_2^μ ($\mu = 0, 1, 2, 3$) are integration constants. These can be further combined to express the x_i as functions of t :

$$x_i(t) = a_i F_T(t) + b_i, \quad (6)$$

$$a_i = c_1^i / c_1^0, \quad (6)$$

$$b_i = c_1^i c_2^0 + c_2^i. \quad (7)$$

For a stationary particle, $a_i = 0$, for all i . For a photon propagating in the x_1 -direction, say, $a_1 = c$, and $a_2 = a_3 = 0$. For these values of the a_i , the line element is trivially seen to vanish.

We now note that a velocity reversion, marked by $dx_i/dt = 0$, occurs whenever $\dot{F}_T(t) = 0$. The periodicity of $F_T(t)$ further guarantees that this condition is fulfilled at a minimum of 2 points in every cycle, the corresponding times being *independent* of the values of a_i and b_i . In other words, each reversion occurs at the *same* value of t on *every* geodesic. These reversions are thus clearly of the Loschmidt kind.

What is more,

- The proper time, $\tau = F_T(t)$, reverses at these points as well, making moments of velocity reversion coincide with those of T-reversal. Thus τ , in contrast to the coordinate time t , does *not* increase monotonically within the interval $[-T/2, T/2]$. Rather, in the simplest case of only 2 points of reversion, t_1 and t_2 say, it increases from t_1 to t_2 , and decreases over the rest of the cycle.
- The line element becomes purely space-like at the points of velocity reversion. This is reflected in the behaviour of the light cone, whose semi-angle, $\alpha = \arctan \dot{F}_T(t)$, vanishes at these points, making the entire space-time *momentarily* space-like.
- Each point of the periodic Minkowski space-time has a horizon of radius, $r = cF_T^{max}$ (where F_T^{max} is the maximum value of F_T): No particle — massive or massless — can go to, and no information can come from, *any* point beyond the horizon. Our metric thus acts like/produces a gravitational potential well, confining all particles and photons to within this horizon. This situation is analogous to that occurring within the Schwarzschild radius of a black-hole: Photons moving radially outwards are dragged back. Our space-time is of course singularity-free but nonetheless the escape velocity is infinite.

Thus if F_T is of the form given in Eq.(2), with T set to 2π for simplicity, $\tau = F_{T=2\pi}(t) = \sin t$, $\alpha = \arctan \cos t$, and the horizon

radius $r = c$. The time cycle extends from $-\pi$ to $+\pi$, and velocity reversions occur at $\pm\pi/2$. The proper time τ increases, as a function of t , from $-\pi/2$ to $+\pi/2$, and decreases in the remaining part of the cycle. The light cone contracts from a semi-opening angle of $-2 \times \arctan 1$ at $t = -\pi \equiv +\pi$ to 0 at $t = -\pi/2$, reopens (in the opposite direction, so to say) out to an angle of $+2 \times \arctan 1$ at $t = 0$, closes again to 0 at $t = \pi/2$ and finally returns to its initial value at $t = \pi$.

All this is for non-interacting particles. What happens if we turn interactions on? The question is difficult to answer in general. However, if we take the interactions to be point-like, a particle would simply switch from one geodesic to another, each time it undergoes an interaction. Subsequent to a moment of velocity reversion, all particles would retrace their paths and collisions, and undergo recurrence exactly as non-interacting particles would.

3.2 A Simple Generalization

It is not difficult to construct more general universes with an S^1 -topology of time. For example, a fairly rich class of recurrence metrics is obtained if we make the spatial components cyclic as well (with the Universal period, T). The corresponding line element is then of the form,

$$ds^2 = c^2 dF_T(t)^2 - dF_T^1(t, x_1)^2 - dF_T^2(t, x_2)^2 - dF_T^3(t, x_2)^2 \quad (8)$$

where the functions, F_T^i ,

- Have dimensions of length,
- Are periodic in $[-T/2, T/2]$,
- Satisfy $\lim_{T \rightarrow \infty} F_T^i(x_i) = x_i, i = 1, 2, 3$.

It can be readily verified that the above metric yields a vacuum solution of the Einstein field equations. Moreover, the t -component of the geodesic equations is same as for the periodic Minkowski metric. The spatial components are, however, much more complicated and are, in fact, not integrable for arbitrary F_T^i .

Since these technically more involved solutions are well outside the primary focus of this paper, we shall not pursue them any further at this point.

4 Summary and Discussion

To summarize, we have examined a new class of vacuum solutions to Einstein's equations, corresponding to (recurrence) Universes with an S^1 -topology of time. Each point of these space-times lies on a geodesic CTC. In contrast to say the Godel and von Stockum universes, which also have CTCs passing through all points, recurrence Universes are free of rotations and simple enough to permit a detailed analysis of how closure in time actually works.

Our analysis has revealed that in recurrence space-times, particle systems return to whichever state they started from at the end of a universal cycle of period T , as a result of an even number (per cycle) of reversions in the velocities of all constituent particles. The velocities become zero on a hypersurface characterized by a single value of coordinate time, and reverse without discontinuity. Once they do so, the system begins to revisit the states it had earlier occupied — in reverse chronological order. This means that if the entropy was earlier increasing, as in the mixing of two confined gases initially separated by a diaphragm, it now begins to decrease, i.e. the gases begin to separate out (and *vice versa*). In other words, each reversion results in a reversal of the arrow of time, defined through entropy change.

This adds an unexpected twist to our understanding of how time-reversible equations at a microscopic level produce time-irreversibility on macroscopic scales. Indeed they seem to suggest that the breaking of T-invariance is, in some sense, spontaneous: There actually exist two distinct phases — one of entropy increase and one of entropy decrease but only one of these is realized in any given situation. In Universes with R^1 -time, there is no way of going from one to the other, but on a CTC there is. In fact, one goes continuously from one to the other, without an energy barrier, as in a second-order phase transition.

We have also found that each reversion is marked by a reversal in the direction of proper time with respect to the time coordinate, parametrizing the temporal S^1 . We have, in turn, traced this to the fact that proper time is periodic (with period T) and smooth, on all CTCs. Indeed, to be so, as a function of the linearly increasing coordinate time, it must have an even number of turning-points on every CTC, each of which must, therefore, contain regions of negative proper time evolution.

This is true, as briefly mentioned, not merely for the CTCs of recurrence space-times but for CTCs in general. In the Gödel universe, for example, all time- and light-like geodesics are bounded by a horizon [30], which is however pierced by CTCs. Negative time travel occurs as one crosses this horizon (with the help, necessarily, of a large non-gravitational force). In wormhole space-times, CTCs thread the wormhole and one goes backwards in time while traversing the latter's throat. In the rotating Kerr, Tomimatsu and von Stockum solutions, the $g_{\phi\phi}$ -component of the metric changes sign for certain values of the coordinates, making ϕ time-like. If we now move in the $-\phi$ -direction, we begin moving backwards in time. The situation in a Gott universe (where we have a deficit wedge angle between the two halves into which the plane passing through two parallel moving cosmic strings divides 3-space) is more complicated. The CTCs of this space-time pass through the deficit wedge region and it is here that negative time evolution occurs.

How does the simultaneous occurrence of negative proper time evolution and entropy decrease tie in with our psychological sense of time? We know that processes evolve locally according to proper time, that they must always be timed one against another, and that psychological time results from watching irreversible processes in the backdrop of simple, reversible oscillatory ones. (These are incidentally not affected by the universal velocity reversion, or rather, they keep undergoing reversions on a much shorter but commensurate time-scale). Thus if each conventionally irreversible process reverses, the sensation will be one of moving backwards in time.

This assumes however that the observer's memory, as a system, is not being continuously reset in the process. The universal evolution of *all* systems towards states they earlier occupied suggests that this assumption should probably be revised.

If we do this, then at $t_2 + \Delta t$, in a cycle containing reversions at t_1 and t_2 , with entropy increasing in $[t_1, t_2]$ and decreasing in $[t_2, t_1]$, all systems, *including the observer's memory* will revert to the states they occupied at $t_2 - \Delta t$. But at $t_2 - \Delta t$, the information stored in the observer's memory relates exclusively to what occurred in $[t_1, t_2 - \Delta t]$. This means that in evolving forward in t , from t_2 to $t_2 + \Delta t$, all memory of what occurred between $t_2 - \Delta t$ and $t_2 + \Delta t$ is effaced. In other words, at every point in the cycle, the observer is aware only of what has occurred in his conventional past, and if proper time evolves backwards, this past keeps shrinking. This moreover suggests that

entropy decrease may actually be unobservable because, at no stage, is the memory of any state with entropy higher than the present ever retained.

In view of the necessity to retrace previously occupied states, it is tempting to speculate that the resolution of the grandfather paradox lies in keeping careful track of the states of the observer as he moves back in time along a CTC. For he must grow younger as he moves back in time towards his childhood and eventually his conception. As such, he moves *only* into his own past and can never access a point in time prior to his birth. The paradox arises from the erroneous assumption that the observer can move back in time without himself changing in any way.

We conclude by raising some issues, which, while being outside the scope of the present discussion, constitute important directions for future research. Firstly, the cosmology of a universe with S^1 -time is of interest, particularly with a view to putting an observational bound on the value of T . Secondly, the quantum generalizations of arguments given in this paper, at the classical level, are essential for a resolution, among other things, of questions relating to non-unitary evolution on CTCs. Thirdly, it is important, for our understanding of time-machines, to extend everything we have discussed to other spacetimes with CTCs. These and other related topics will be reported on elsewhere.

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