

The Quark Gluon Pion Plasma

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Abstract

While it is commonly believed that there is a *direct* transition from the hadronic to a quark gluon phase at high temperature, it would be prejudicial to rule out a sequence of dynamically generated intermediate scales. Using as guide, an effective lagrangian with unconfined gluons and constituent quarks, interacting with a chiral multiplet, we examine a scenario in which the system undergoes first-order transitions at T_{comp} , the compositeness scale of the pions, at T_χ , the scale for spontaneous chiral symmetry breaking, and at T_c , the confinement temperature.

We find that at current energies, it is likely that the formation temperature of the plasma, $T_0 < T_{comp}$, and that this is therefore a quark gluon pion plasma (QGPP) rather than the usual quark gluon plasma (QGP). We propose some dilepton-related signatures of this scenario.

We know that quarks and gluons are confined as hadrons and chiral symmetry is spontaneously broken. We do not however know if these two phenomena set in at an identical temperature. It is unlikely that the chiral symmetry restoration energy/temperature scale is *lower* than that of confinement, because if it were, hadrons would show parity doubling below the confinement but above the chiral breaking scale. This is not seen either experimentally or in finite temperature lattice simulations. Determining whether it is *higher* even in the simplest case of QCD with a two flavour $SU_2(L) \times SU_2(R)$ chiral symmetry is not straight-forward. Indeed, while there is a bonafide order parameter for chiral symmetry breaking — namely, the mass of the constituent quark — the Wilson loop in the presence of dynamical quarks, is *not* a valid order parameter for confinement — and there are no other known candidates. However, by looking at energy density or specific heat, we can get a fair idea of the change in the number of operational degrees of freedom or particle modes with temperature. The relevant lattice calculations indicate that the drop from the large number of degrees of freedom in the QGP/QGPP phase to a few degrees of freedom in the hadronic one takes place in one broad step in temperature, suggesting that the two transitions are close but not necessarily identical, i.e. that $T_\chi \gtrsim T_c$.

In the same spirit, we could ask whether pions necessarily come apart the moment chiral symmetry is restored, or equivalently, whether the compositeness scale for the pion, T_{comp} , coincides with T_χ . The following analogy with

superconductivity (SC) leads us to conclude that it may not:

In the usual BCS theory, the formation of Cooper pairs at T_{crit} coincides with the appearance of a SC order parameter. This corresponds to $T_{comp} = T_{crit}$, as there are no Cooper pairs at scales above T_{crit} . Here, T_{comp} is the compositeness scale for Cooper pairs, which are normally described as undergoing momentum space pairing, the interelectron distance being much less than the size of a Cooper pair. This is indeed what happens for a weak pairing interaction. For a much stronger interaction, Cooper pairs may actually form by real space pairing. The Cooper pair size is then smaller than the interelectron distance and it is possible that, even when SC is lost, i.e. $T > T_{crit}$, pairs continue to exist simply as bound states.

Evidence for the related precursor phenomena for the chiral transition had been suggested long ago [14]. Recently, there have been several calculations of the QCD counterpart of the non-condensed, paired, pseudogap phase of high T_c superconductivity [15], which suggest that the strong pairing situation is likely for the strong interactions.

We shall accordingly take $T_{comp} > T_\chi$, and argue that heavy-ion collisions are the ideal platform for testing this assumption. In doing so, we shall use the following $SU_2(L) \times SU_2(R)$ effective lagrangian, in which the quarks are coupled to a chiral multiplet, $[\pi, \sigma]$ [2, 3, 4], to keep track of particle modes at various scales:

$$L = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a - \sum \bar{\psi} (D + g_y(\sigma + i\gamma_5 \vec{\tau} \vec{\pi})) \psi - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \text{const} \quad (1)$$

This has chiral symmetry-breaking but no confinement. The masses of the scalar/pseudoscalars and fermions, obtained by minimizing the potentials above, are $\mu^2 = -\lambda < \sigma >^2$ and $m_\sigma^2 = 2\lambda < \sigma >^2$ respectively. This theory reproduces several broad features of the strong interaction at the mean field level [5, 6], indicating that it is, perhaps, a reasonable guide to the physics at intermediate scales. More specifically,

1. It provides a nucleon, which is realized as a soliton with quarks bound in a skyrmion configuration [3, 5]. Such a nucleon:
 - (a) Gives a natural explanation for the ‘Proton spin puzzle’: Quarks in the background fields are in a spin, ‘isospin’ singlet state in which the quark spin operator averages to zero. If we collectively quantize the soliton to get states of good spin and isospin, the quark spin operator picks up a small, non-zero expectation value [7].
 - (b) Seems to naturally produce the Gottfried sum rule [8].
 - (c) Yields, from first principles (*albeit* with some drastic QCD evolution), a set of structure functions for the nucleon which are close

to the experimental ones [9].

2. As a finite temperature field theory, it yields screening masses that match with those obtained from the lattice simulation of finite temperature QCD with dynamical quarks [10].
3. It gives a consistent equation of state for strongly interacting matter at all densities [11, 5].

This lagrangian and the above discussion on temperature scales lead us to consider the following scenario:

1. For $T > T_{comp}$, we have gluons and massless current quarks, in a pure QGP phase.
2. For $T_{comp} > T > T_\chi$, we have a QGPP phase for which the particle mode count is (i) 16 modes from 8 massless gluons, (ii) 24 from 2 flavors of massless current quarks and anti-quarks, and (iii) 3 degenerate pions, which are present, in this temperature range, in the form of non-Goldstone-boson bound states, which we *assume* to be massless. (We neglect the σ , which would otherwise contribute one more mode).

The operational degrees of freedom in this phase are then 40 — three more than in the QGP phase. This increase may be visible as a slight bump on an ‘Energy Density’ *vs* ‘Temperature’ plot, in a high resolution lattice simulation.

3. For $T_\chi > T > T_c$, chiral symmetry is spontaneously broken and the degrees of freedom are (i) gluons, (ii) constituent quarks, which acquire a constituent mass of $300 - 400$ MeV from the spontaneous breaking of χ -symmetry and are, as a result, Boltzman suppressed given that $T_\chi \simeq T_c$ is expected [1] to be $O(200MeV)$, (iii) pions as Goldstone bosons, and (iv) the σ , which also acquires a mass and may be neglected.

The effective degrees of freedom are thus 19, at this stage.

4. For $T_c > T$, quarks are confined, and the only massless modes left are the 3 Goldstone bosons/pions.

We can get an estimate of T_{comp} by examining the behaviour of the above Lagrangian as a function of energy. We find that at a certain scale, both the wave function renormalization constant, Z , for the π and σ , *and* the quartic scalar interaction, vanish, leaving us with a Yukawa and a scalar mass term. The vanishing of Z , or equivalently, of the kinetic term for the scalar and pseudoscalars, means that these are no longer dynamical degrees of freedom, i.e. they have ceased to exist as composite entities. Eliminating them, via their field equation, leaves behind a four-fermi term which gets weaker with increasing energy. While these results, gleaned using perturbative RNG, cannot be used all the way out to where Z goes to zero (since the Yukawa coupling has a Landau singularity there), they can perhaps give a reasonable estimate for the compositeness scale. This works out to be around $700 - 800$ MeV [12], well above the temperatures attained in the present-day heavy

ion colliders. This implies that the initial state produced by these is very probably a QGPP.

To investigate this possibility further, we adapt to our scenario, the considerations of [1], which gives a very transparent and simple treatment of the underlying physics. In particular, following [1], we treat each of our transitions as first order, with each producing a mixed phase in which the number of degrees of freedom changes continuously, as one phase gives way to the other.

A plasma which thermalizes at a temperature, T_0 , where $T_{comp} < T_0 < T_\chi$, a proper time, t_0 , after nuclear impact, then evolves as follows:

1. It Bjorken expands and cools [13], reaching T_χ at time, $t_{x1} = (T_0/T_\chi)^3 t_0$.
2. At T_χ , it undergoes a first-order transition from the quark-gluon-pion to the chirally broken, gluon-pion phase, from t_{x1} to $r_1 t_{x1}$, where $r_1 = 40/19$ is the ratio of the degrees of freedom in the initial and final phases.

N.B. If we assume that chiral restoration and deconfinement occur simultaneously at $T_\chi = T_c$, the quark phase lasts in the mixture from time $t_1 = t_{x1}$ to $t_2 = r t_1$ (where $r = 37/3$), which is substantially longer.

3. Having passed to the chirally-broken phase, the plasma undergoes a

second Bjorken expansion and thereby cools from T_χ to T_c . Since $T_\chi \simeq T_c$, this part of the evolution may, however, be neglected.

4. At T_c , we get a mixture of the gluon-pion and the purely hadronic confined phases, which lasts from $r_1 t_{x1}$ to $t_{x2} = r_2 r_1 t_{x1}$, where r_2 is $19/3$.
5. Finally the pion phase expands from T_c to T_f , the freezeout temperature [1], at which pions loose thermal contact with one another. This phase begins at time t_{x2} and ends at $t_f = (T_c/T_f)^3 t_{x2}$.

The basic changes in this scenario when compared to that of [1] are that, (i) the pionic phase starts much earlier, at the initial temperature, T_0 , and not after confinement at T_c and (ii) the quark phase is suppressed above T_c and is thus shortened.

To see how these changes are reflected in the number of dileptons emitted by the plasma, we note that these come from $\pi^+\pi^-$ and $q\bar{q}$ annihilations. The cross-section for $q\bar{q} \rightarrow l^+l^-$, summed over the spin, flavour and color of all quarks is

$$\sigma_q[M] = F_q \frac{4\pi}{3} \left(\frac{\alpha}{M}\right)^2 \sqrt{1 - 4\left(\frac{m_l}{M}\right)^2} \left(1 + 2\left(\frac{m_l}{M}\right)^2\right) \quad (2)$$

where the numerical factor, $F_q = 20/3$, if we include just the u and d quarks in the flavor sum.

The cross-section for $\pi^+\pi^- \rightarrow l^+l^-$ likewise is,

$$\sigma_\pi[M] = F_\pi[M](4\pi/3)(\alpha/M)^2\sqrt{1-4(m_l/M)^2}(1+2(m_l/M)^2)\sqrt{1-4(m_\pi/M)^2} \quad (3)$$

where the pion form factor is given by,

$$F_\pi[M] = m_\rho^4/((m_\rho^2 - M^2)^2 + m_\rho^2\Gamma_\rho^2) + (1/4)m_{\rho'}^4/((m_{\rho'}^2 - M^2)^2 + m_{\rho'}^2\Gamma_{\rho'}^2) \quad (4)$$

The masses and decay widths of ρ and ρ' resonances are $m_\rho = 775$ MeV, $m_{\rho'} = 1.6$ GeV, $\Gamma_\rho = 155$ MeV, $\Gamma_{\rho'} = 260$ MeV respectively.

The rate for producing pairs, per unit space-time, with invariant mass squared, M^2 , and transverse energy, $E_T = \sqrt{p_T^2 + M^2}$, where p_T is the momentum transverse to the beam axis, is (for $a = q, \pi$),

$$\frac{dN_a}{d^4x dM^2 dE_T} = \frac{\sigma_a[M]M^2}{4(2\pi)^4} \left(1 - 4\frac{m_a^2}{M^2}\right) E_T K_0(E_T/T) = A_a[M] E_T K_0(E_T/T) \quad (5)$$

While using Bjorken's model, it is expedient to write $d^4x = d^2x_T dy dt$ where t is the proper time and y , the rapidity, of the fluid element. For central collisions of equal mass nuclei $d^2x_T = \pi R_A^2$, where R_A is the nuclear radius.

We can further perform the t -integral to get:

$$\frac{dN_q^{MT}}{dy dM^2 dE_T} = \pi R_A^2 A_q(M) \left\{ 3E_T^{-5} T_0^6 t_0^2 (G[E_T/T_0] - G[E_T/T_c]) + E_T K_0[E_T/T_c] (1/2)(r_1 - 1)t_{x1}^2 \right\}, \quad (6)$$

for the quark channel in the multiple transitions (MT) scenario. The first term in braces comes from integrating along the cooling curve $t = (T_0/T)^3 t_0$

from T_0 to T_c and the second, from the coexistence line at T_χ , which we approximate as being equal to T_c . The shortening of the quark phase is reflected in the presence here of $r_1 = 40/19$, as opposed to $r = 37/3$ for the single transition (ST) at $T_\chi = T_c$ case.

The corresponding rate for $\pi^+\pi^-$ -annihilations is likewise,

$$\begin{aligned} \frac{dN_\pi^{MT}}{dydM^2dE_T} = \pi R_A^2 A_\pi(M) \left\{ 3E_T^{-5} T_0^6 t_0^2 (G[E_T/T_0] - G[E_T/T_c]) \right. \\ \left. + 3E_T^{-5} T_c^6 t_{x2}^2 (G[E_T/T_c] - G[E_T/T_f]) + E_T K_0[E_T/T_c] (t_{x2}^2 - t_{x1}^2)/2 \right\} \end{aligned} \quad (7)$$

where,

$$G[x] = x^3(8 + x^2)K_3[x] \quad (8)$$

and $K_i[x]$ are the modified Bessel functions.

In this case, the first term in braces (absent altogether for the ST case) results from the cooling of pions in the QGPP, from T_0 and T_c , and the second, from their cooling in the hadronic phase, from T_c to T_f . The latter coincides with the ST result if $t_{x2} \rightarrow t_2$. The last term comes from integrating over time, the volume fraction, $f(t)$, of the pion phase, on the coexistence line $T_\chi \simeq T_c$. Since pions are present both above and below each of the coexistence lines at T_χ and T_c , $f(t) = 1$ for both these transitions. In the approximation, $T_\chi = T_c$, the integral is then simply,

$$\int_{t_{x1}}^{t_{x2}} t dt = (t_{x2}^2 - t_{x1}^2)/2. \quad (9)$$

The analogous integral in the ST case is, by contrast, $(r - 1)rt_1^2/2$.

We note that these differential rates further integrate over E_T to the closed-form expressions:

$$\frac{dN_q^{MT}}{dydM^2} = \pi R_A^2 A_q(M) T_0^2 t_0^2 \left\{ 6(T_0/M)^{-4} (H[M/T_0] - H[M/T_c]) \right. \\ \left. + (M/T_c)(T_0/M)^{-4} K_1[M/T_c](r_1 - 1) \right\} \quad (10)$$

where,

$$H[x] = x^2(8 + x^2)K_0[x] + 4x(4 + x^2)K_1[x] \quad (11)$$

and we have translated ratios of times into ratios of temperatures. Similarly,

$$\frac{dN_\pi^{MT}}{dydM^2} = \pi R_A^2 A_\pi(M) T_0^2 t_0^2 \left\{ 6(T_0/M)^4 (H[M/T_0] - H[M/T_c]) + 6(r_1 r_2)^2 (T_0/M)^4 \right. \\ \left. (H[M/T_c] - H[M/T_f]) + (M/T_c)(T_0/T_c)^4 K_1[M/T_c] [(r_1 r_2)^2 - 1] \right\} \quad (12)$$

Finally, the differential rates, $dN_{tot}^{MT}/dydM^2 dE_T$ and $dN_{tot}^{MT}/dydM^2$ for the *total* number of pairs, are obtained by summing the right hand sides of Eqs.(6) and (7), and Eqs.(10) and (12) respectively.

We can now compare our scenario, in quantitative detail, with that of Ref.[1]. For ease of comparison, we use the same parameter-values: $t_0 = 1fm/c$, and $\pi R_A^2 = 127fm^2$.

- In Fig.1, we plot $dN_{tot}^{MT}/dydM^2 dE_T$, $dN_q^{MT}/dydM^2 dE_T$ and $dN_\pi^{MT}/dydM^2 dE_T$ vs M , for the combinations of T_0 and T_c used in Ref.[1] but at values

of E_T , for which (in the first two cases) the ST pion peaks, coming from annihilation via the ρ and ρ' resonances, are largely extinguished. We note that in the MT scenario, the rate displayed reduces in magnitude as E_T increases, but the pion peaks survive and stay well above the $q\bar{q}$ contribution. This results from pions being created when the plasma forms, as opposed to when it hadronizes. It is consistent with the observation in [1] that peak extinction at large E_T and the initial temperature of the pions are distinctly correlated: the smaller this temperature, the smaller the E_T at which the peaks get extinguished.

- We note that an enhancement in in the production of large E_T *dileptons*, owing to pions being available at higher temperatures, will inevitably be accompanied by an enhancement in the production of large p_T *pions*, a feature highlighted by Schucraft [16] long ago. We can add that pions present at the boundary of the initial plasma, will, being colourless, have no problem escaping from the interaction region. The availability of pions at higher temperatures is, incidentally, supported by the observation that the HBT size of the initial pion source is smaller than if pions formed only after hadronization [17, 18].
- We would intuitively expect the difference between the two scenarios to depend (among other factors) on the time spent by the system in the QGPP phase. This would in turn be determined by the initial temperature, T_0 . In Fig.2, we accordingly plot the ratio of $dN_{tot}/dydM^2$

for the MT and ST scenarios *vs* T_0 over the range $T_\chi \leq T_0 \leq T_{comp}$, for several values of M . We find that the MT rate is 20 – 30% higher, when M is not too far from the ρ , ρ' resonances, but drops sharply as M increases and the drooping pion form factor begins to cut down the pion contribution.

- We have noted that quarks acquiring constituent masses larger than T_χ and decoupling at the chiral phase transition, reduces the lifetime of the quark phase *above* T_c . This, in turn, brings down the $q\bar{q} \rightarrow l^+l^-$ rate for small M and E_T . In Fig.1, the difference is visible but only when $T_c = 240MeV$ and $T_0 = 250MeV$. In the total rate, the effect is swamped by the pion contribution for those values of M for which the pion form factor is appreciable. However, from Fig.3, we see that for slightly larger values of M , the pion contribution disappears and the total differential rate $dN_{tot}/dydM^2$ reduces to just the quark contribution, for both the ST and MT cases. Determining the rate in this M -window would then give an indication of whether a chiral transition has occurred close to but distinctly above T_c .
- The behaviour of these curves as functions of T_0 can be further understood as follows: In the ST scenario, a higher T_0 does not change the pion contribution to dilepton production. However, it clearly increases the time spent by the quarks in the cooling phase and thus enhances the quark contribution to dileptons, pushing the crossover

to the quark dominated regime to smaller M . For the MT scenario, both the quark and pion contributions to dileptons are enhanced and so the crossover to the quark-dominated regime is not expected to change much (though it may still move slightly towards smaller M , due to steep fall in the pion form factor). For both scenarios, the cooling term in the quark contribution increases in magnitude with T_0 while the Maxwell co-existence term (which accounts for the difference between the two scenarios) remains unchanged. Thus as T_0 increases, the ST and MT quark contributions approach each other.

- Finally, in Fig.4, we plot the ratio of $dN_{tot}/dydM^2$ for the MT and the No Transition (NoT) scenarios *vs* T_0 . The latter corresponds to there being no phase transition, i.e. to the fireball's cooling in the pure hadronic phase. It is seen that this ratio is enormous.
- We conclude that if the dilepton rates increase dramatically, a phase transition has probably occurred. If the pion peaks in $dN_{tot}/dydM^2dE_T$ *vs* M survive to high values of E_T , the plasma has a good chance of being a QGPP. If the peaks do *not* survive, but rather reduce to a flat line, we are quite likely seeing a quark phase. The actual value of this constant rate, for the window in M discussed above, will then decide whether the transition to the confined hadronic phase occurred directly or through a chirally broken phase.

In summary, we have argued that there *are* reasons to believe that a QPG may form in heavy-ion collisions, as a result of several, as opposed to a single, phase transition. We have explored this possibility on the basis of some simple assumptions. We have found that this scenario pushes to higher temperatures the advent of the QGP, but opens up, in return, the exciting possibility of a QGPP, containing pions as non-Goldstone-boson bound states, well above T_χ . We have further investigated experimental signatures based on dilepton production, which would help distinguish the single- from the multiple-transition scenario. Making this distinction would shed light not merely on how the QGP forms but on a range of assumptions which lie at the very core of our understanding of strong interaction dynamics.

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Figure Captions

Figure 1 Plots of $dN_{tot}^{MT}/dydM^2dE_T$ vs M for three (T_0, T_c) -combinations.

For the values of E_T in Fig.1.a and Fig.1.b, the pion peaks in the single transition rate are essentially non-existent. Also plotted are the quark contributions for both the MT (thick line) and ST (thin line) scenarios. These can be distinguished only for $T_0 = 250MeV$ and $T_c = 240MeV$. The MT quark contribution is lower because of the shortening of the quark phase.

Figure 2 Plots of the ratio of the $dN_{tot}/dydM^2$ rate *vs* T_0 for the MT and ST scenarios. For the *lower* M -values considered in these graphs, the pion form factor is appreciable. It seen that the MT rate is distinctly higher than the ST rate.

Figure 3 Plots of the ratio of the $dN_{tot}/dydM^2$ rates for the MT and ST scenarios *vs* T_0 . The pion contribution is essentially absent for the *higher* M -values chosen. Note that the MT rate is now *lower* than the ST rate. The dashed lines represent the ratio of just the quark contributions. Quarks contribute fewer dileptons in the MT scenario, where the quark phase is shortened by the occurrence of the chiral transition.

Figure 4 The ratios of the MT rate to the rate if no transition occurred at all.

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